1 Show the following identities:

1 a

$$\int \frac{dz^*}{(2\pi i)^{1/2}} \frac{dz}{(2\pi i)^{1/2}} \, e^{-z^* b z} = \frac{1}{b}$$

$b \in \mathbb{R}$

$\alpha, z \in \mathbb{C}$

1 b

$$\int \frac{dz^*}{(2\pi i)^{1/2}} \frac{dz}{(2\pi i)^{1/2}} \, z^* z \, e^{-z^* b z} = \frac{1}{b^2}$$

1 c

$$\int \frac{d\alpha \, d\alpha^*}{2\pi i} \, e^{-\alpha\alpha^*} \, \underbrace{|\alpha\rangle\langle\alpha^*|}_{\text{(coherent states)}} = \hat{1}$$

2 Consider the forced harmonic oscilator written in harmonic phase space (pages 33 to 38 of my lecture notes, on the website). Work out the missing steps, in particular:

2 a Show that the classical solutions in 36.2 are really solutions

2 b Prove that:

$$\tilde{S}[\alpha(t), \alpha^*(t); \gamma(t), \bar{\gamma}(t)] = \tilde{S}[\alpha_{cl}(t), \alpha^*_{cl}(t); \gamma(t), \bar{\gamma}(t)] + \tilde{S}[\tilde{\alpha}(t), \tilde{\alpha}^*(t); 0, 0]$$

2 c Prove eq. 36.3

3 Given the generating functional for the forced harmonic oscilator (eq. 31.2 in my notes):

$$Z[J] = N \, e^{-\frac{1}{2} J \cdot \Delta \cdot J + i q_{cl}(0) \cdot J}$$

obtain the 4-point function: $G_4(t_1, t_2, t_3, t_4)_J$

and then specialize it to the case without sources: $G_4(t_1, t_2, t_3, t_4)_0$
(simple harmonic oscillator)